OCR
RECOGNISING ACHIEVEMENT

Wednesday 16 May 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715760612

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Some differential equations of the form

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = f(x)$$

are to be solved.

First consider the case $f(x) = x^2$.

(i) Find the general solution for y in terms of x. **[9]**

(ii) Find the particular solution subject to the conditions $y = 0$, $\frac{dy}{dx} = 0$ when $x = 0$. **[5]**

Now consider the case $f(x) = e^{-3x}$.

(iii) Explain why neither ae^{-3x} nor axe^{-3x} will be a particular integral for the differential equation. **[1]**

(iv) State an appropriate form for a particular integral and hence find the general solution. **[7]**

(v) State with reasons whether it is possible to have particular solutions for which

(A) y is positive for all values of x,

(B) y is negative for all values of x. **[2]**

2 A parachutist of mass m kg falls vertically from rest. After she has fallen x m, her speed is v m s^{-1}. The forces acting on her are her weight and a resistance force of magnitude mkv^2 N, where k is a constant.

(i) Show that her motion is modelled by the differential equation

$$v\frac{dv}{dx} = g - kv^2$$

and solve this to show that $v^2 = \frac{g}{k}(1 - e^{-2kx})$. **[8]**

(ii) Given that her terminal speed is 55 m s^{-1}, calculate k. **[1]**

When her speed is 54 m s^{-1}, she opens her parachute. The motion is now modelled by assuming that the magnitude of the resistance force instantaneously changes to $0.1mgv$ N. The time from the parachute opening is t seconds.

(iii) Formulate and solve a differential equation to find v in terms of t. **[8]**

(iv) Calculate the time it takes for her speed to reduce to 12 m s^{-1}. **[1]**

(v) Calculate the distance she falls from the point at which she opens her parachute to the point at which her speed is 12 m s^{-1}. **[6]**

3 The differential equation $x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y = x^3 \sin x$ is to be solved.

(i) Find the general solution for y in terms of x. **[8]**

(ii) Find the particular solution subject to the condition $y = 0$ when $x = \pi$. Sketch the solution curve for $0 \leqslant x \leqslant 4\pi$. **[5]**

Now consider the differential equation $x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y^2 = 0$.

(iii) Find the general solution for y in terms of x. **[5]**

Now consider the differential equation $x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y^2 = x^3 \sin x$.

This is to be solved numerically using Euler's method. The algorithm is given by
$x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$ with $(x_0, y_0) = (3.14, 0)$.

(iv) Use a step length of 0.01 to estimate y when $x = 3.16$. **[5]**

(v) How could this estimate be improved? **[1]**

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -2x - y + 6,$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = x - 2y + 7,$$

are to be solved.

(i) Eliminate y to obtain a second order differential equation for x in terms of t. Hence find the general solution for x. **[12]**

(ii) Find the corresponding general solution for y. **[3]**

Initially $x = 7$ and $y = 0$.

(iii) Find the particular solutions. **[4]**

As $t \to \infty$, $\dfrac{y}{x} \to k$.

(iv) State the value of k and show that $y = kx$ for infinitely many values of t. **[5]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1(i)	

1(ii)	
1(iii)	

1(iv)	

1(v)(*A*)	
1(v)(*B*)	
2(i)	
	(answer space continued overleaf)

2(i)	(continued)
2(ii)	
2(iii)	

2(iii)	(continued)
2(iv)	
2(v)	
	(answer space continued overleaf)

2(v)	**(continued)**
3(i)	

3(i)	(continued)
3(ii)	

3(iii)	
3(iv)	

3(iv)	(continued)
3(v)	
4(i)	
	(answer space continued overleaf)

4(i)	(continued)

4(i)	(continued)

4(ii)	
4(iii)	

4(iii)	(continued)
4(iv)	
	(answer space continued overleaf)

4(iv)	(continued)

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4758:** Differential Equations

Mark Scheme for June 2012

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

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Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions: GCE Mathematics (Mechanics)

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (eg lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

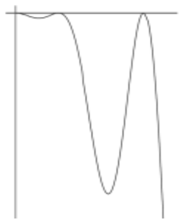
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance
1	(i)		$\lambda^2+6\lambda+9=0$ $\lambda=-3$ (repeated) $y=\left(A+Bx\right)\mathrm{e}^{-3x}$ $y=ax^2+bx+c$ $y'=2ax+b,\ y''=2a$ $2a+6(2ax+b)+9(ax^2+bx+c)=x^2$ $9a=1$ $12a+9b=0$ $2a+6b+9c=0$ $a=\frac{1}{9},\ b=-\frac{4}{27},\ c=\frac{2}{27}$ $y=\frac{1}{9}x^2-\frac{4}{27}x+\frac{2}{27}+(A+Bx)\mathrm{e}^{-3x}$	M1 A1 F1 B1 M1 M1 M1 A1 F1 **[9]**	 Differentiate and substitute Compare at least two coefficients Solve for at least two unknowns Non-zero PI + CF with two arbitrary constants
1	(ii)		$x=0, y=0\Rightarrow A=-\frac{2}{27}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{9}x-\frac{4}{27}+B\mathrm{e}^{-3x}-3(A+Bx)\mathrm{e}^{-3x}$ $x=0,\ y'=0\Rightarrow 0=-\frac{4}{27}+B-3A$ $\Rightarrow B=-\frac{2}{27}$ $y=\frac{1}{9}x^2-\frac{4}{27}x+\frac{2}{27}-\frac{2}{27}(1+x)\mathrm{e}^{-3x}$	M1 M1 F1 M1 A1 **[5]**	Use condition Differentiate using product rule FT only c from (i) Use condition cao
1	(iii)		Both appear in CF	B1 **[1]**	Or any clear statement or reason
1	(iv)		$y=\mu x^2\mathrm{e}^{-3x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2\mu x\mathrm{e}^{-3x}-3\mu x^2\mathrm{e}^{-3x}$	B1 M1	Allow for $y=\lambda x^3\mathrm{e}^{-3x}+\mu x^2\mathrm{e}^{-3x}$ Differentiate twice using product rule

Question			Answer	Marks	Guidance
			$\frac{d^2y}{dx^2} = 2\mu e^{-3x} - 12\mu x e^{-3x} + 9\mu x^2 e^{-3x}$	A1	
			in DE: $e^{-3x}\left(2\mu - 12\mu x + 9\mu x^2 + 12\mu x - 18\mu x^2 + 9\mu x^2\right) = e^{-3x}$	M1	Substitute into DE
			$\Rightarrow 2\mu = 1 \Rightarrow \mu = \frac{1}{2}$	M1	Compare coefficients
				A1	Correct PI correct working only
			$y = \left(A + Bx + \frac{1}{2}x^2\right)e^{-3x}$	F1 **[7]**	Non-zero PI + CF with two arbitrary constants
1	**(v)**	*(A)*	$(e^{-3x} > 0)$, quadratic has +ve coefficient of x^2 so can have $y > 0$ for all x for suitable A and B	B1	FT for any three term quadratic Reasonably complete explanation or show for a specific example
1	**(v)**	*(B)*	would need quadratic < 0 for all x, but for large x it will be positive	B1 **[2]**	FT for any three term quadratic Reasonably complete explanation For both marks to be awarded in (v), $e^{-3x} > 0$ must be stated
2	**(i)**		$mv\frac{dv}{dx} = mg - mkv^2$	M1	N2L 3 terms, allow sign errors and any form for accn, including a
			$v\frac{dv}{dx} = g - kv^2$	E1	
			$\int \frac{v}{g - kv^2}\, dv = \int dx$	M1*	Separating variables. Integrating factor attempt gets zero
			$-\frac{1}{2k}\ln\lvert g - kv^2\rvert = x + c_1$	A1	LHS
				A1	RHS (including constant on one side)
			$g - kv^2 = Ae^{-2kx}$	M1dep*	Rearrange, dealing properly with constant
			$x = 0,\ v = 0 \Rightarrow A = g$	M1dep*	Use condition
			$v^2 = \frac{g}{k}\left(1 - e^{-2kx}\right)$	E1 **[8]**	
2	**(ii)**		$55^2 = \frac{g}{k} \Rightarrow k = \frac{g}{3025}$	B1 **[1]**	Or 0.00324: cao

Question			Answer	Marks	Guidance
2	(iii)		$m\frac{dv}{dt} = mg - 0.1mgv$	M1	N2L 3 terms, allow sign errors and any form for accn, including a
			$\frac{dv}{dt} = g(1-0.1v)$	A1	
			EITHER		
			$\int \frac{1}{1-0.1v}\,dv = \int g\,dt$	M1	Separating variables
			$-10\ln\|1-0.1v\| = gt + c_2$	A1	LHS
				A1	RHS (including constant on one side)
			$1-0.1v = Be^{-0.1gt}$	M1	Rearrange
			$t=0, v=54 \Rightarrow B=-4.4$	M1	Use condition
			$v = 10 + 44e^{-0.1gt}$	A1	
				[8]	
			OR		
			Integrating factor $e^{0.1gt}$	B1	FT their equation
			$ve^{0.1gt} = 10e^{0.1gt} + A$	M1	Multiply through by IF and integrate both sides
				A1	FT
			$v = 10 + Ae^{-0.1gt}$	M1	Divide through by IF
			$t=0, v=54 \Rightarrow A=44$	M1	Use condition
			$v = 10 + 44e^{-0.1gt}$	A1	
				[8]	
			OR		
			$\frac{dv}{dt} + 0.1gv = g$		
			Auxiliary equation $m + 0.1g = 0$ CF $Ae^{-0.1gt}$	M1 A1	
			PI $v=10$	B1	
			GS $v = 10 + Ae^{-0.1gt}$	M1	
			$t=0, v=54 \Rightarrow A=44$	M1	Use condition
			$v = 10 + 44e^{-0.1gt}$	A1	
				[8]	

Question			Answer	Marks	Guidance
2	(iv)		$t=-\frac{1}{0.1g}\ln\left(\frac{12-10}{44}\right)\approx 3.15$ s	B1 FT **[1]**	FT for v obtained by a correct method
2	(v)		$x=\int v\,\mathrm{d}t=10t-\frac{440}{g}\mathrm{e}^{-0.1gt}+c_3$ $t=0, x=0\Rightarrow c_3=\frac{440}{g}$ $t=3.15\Rightarrow x=74.4$	M1 F1 M1 A1 FT M1 A1 **[6]**	FT attempt to integrate their answer to part (iii) FT for x obtained by a correct method cao
3	(i)		$\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{2}{x}y=x^2\sin x$ $I=\exp\left(-\int\frac{2}{x}\,\mathrm{d}x\right)$ $=\mathrm{e}^{-2\ln x}$ $=x^{-2}$ $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{-2}y\right)=\sin x$ $x^{-2}y=\int\sin x\mathrm{d}x$ $x^{-2}y=-\cos x+A$ $y=x^2\left(-\cos x+A\right)$	M1 M1 A1 A1 M1 B1 A1 F1 **[8]**	Divide both sides by x (NOTE that a MR (eg sin$x$ missing) can earn 7/8) Allow $\pm$ Multiply by their IF and attempt to integrate both sides LHS must be their IF x y + A is needed Divide every term by the multiplier of y (including a constant)
3	(ii)		$0=\pi^2\left(-\cos\pi+A\right)$ $\Rightarrow A=-1$ $y=-x^2(\cos x+1)$	M1 A1	Use condition cao

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td></td><td></td><td></td><td></td><td>B1
B1
B1

[5]</td><td>Oscillations corresponding to their expression
Changing amplitude corresponding to their expression
$y \le 0$ with maxima on x-axis; cusps get B0; cao
Max B1 if MR has made graph simpler</td></tr>
<tr><td>3</td><td>(iii)</td><td></td><td>$\frac{1}{y^2}\frac{dy}{dx} = \frac{2}{x}$
$\int \frac{1}{y^2}\,dy = \int \frac{2}{x}\,dx$
$-\frac{1}{y} = 2\ln x + B$

$y = \frac{-1}{2\ln x + B}$</td><td>M1
M1
A1
A1
A1
[5]</td><td>Attempt to separate variables Attempt at IF, M0 for this part
Integrate both sides
LHS
RHS (including constant on one side)
Any correct form of answer</td></tr>
<tr><td>3</td><td>(iv)</td><td></td><td>$\frac{dy}{dx} = \frac{x^3 \sin x + 2y^2}{x}$
<table><tr><td>x</td><td>y</td><td>y'</td><td>hy'</td></tr><tr><td>3.14</td><td>0</td><td>0.015703</td><td>0.00015703</td></tr><tr><td>3.15</td><td>0.00015703</td><td>–0.083421</td><td>–0.00083421</td></tr><tr><td>3.16</td><td>–0.000677</td><td></td><td></td></tr></table></td><td>B1
M1
A1
A1
A1
[5]</td><td>May be implied by correct values
Use algorithm can get this even if B0
$y(3.15) = 0.000157\ldots$ Agreement to 3sf
–0.08342… Agreement to 3sf Award for -0.000834
–0.000676…Agreement to 3sf. ONLY award if this is the FINAL answer</td></tr>
<tr><td>3</td><td>(v)</td><td></td><td>Reduce step length (and carry out more steps)</td><td>B1
[1]</td><td>No contradictions</td></tr>
</table>

Question			Answer	Marks	Guidance
4	**(i)**		$y = -\dot{x} - 2x + 6$	M1	
			$\dot{y} = -\ddot{x} - 2\dot{x}$	M1	
			$-\ddot{x} - 2\dot{x} = x + 2\dot{x} + 4x - 12 + 7$	M1	Substitute for y
				M1	Substitute for $\dot{y}$
			$\ddot{x} + 4\dot{x} + 5x = 5$	A1	cao
			AE $\lambda^2 + 4\lambda + 5 = 0$	M1	
			$\lambda = -2 \pm j$	A1	cao
			CF $e^{-2t}(A\cos t + B\sin t)$	M1	CF for complex roots
				B1	CF for their roots
			PI $x = \frac{5}{5} = 1$	B1 B1	Appropriate form of PI Correct PI
			GS $x = 1 + e^{-2t}(A\cos t + B\sin t)$	F1	Non-zero PI + CF with 2 arbitrary constants
				[12]	
4	**(ii)**		$\frac{dx}{dt} = -2e^{-2t}(A\cos t + B\sin t) + e^{-2t}(-A\sin t + B\cos t)$	M1	Differentiate using product rule
			$y = -\dot{x} - 2x + 6 \quad = 4 + e^{-2t}(A\sin t - B\cos t)$	M1 A1	Substitute cao
				[3]	
			Alternative method: $\frac{dy}{dt} = 1 + e^{-2t}\left(A\cos t + B\sin t\right) - 2y + 7$		
			$\frac{dy}{dt} + 2y = 8 + e^{-2t}\left(A\cos t + B\sin t\right)$	M1	Substitute for x in $\frac{dy}{dt} = x - 2y + 7$ and arrange in correct IF form
			$ye^{2t} = \int (8e^{2t} + A\cos t + B\sin t)dt$	M1	Find IF, multiply both sides by IF and show intention to integrate
			$y = 4 + e^{-2t}(A\sin t - B\cos t)$	A1	
				[3]	
4	**(iii)**		$7 = 1 + A$	M1	Use condition on a GS
			$0 = 4 - B$	M1	Use condition on a GS
			$x = 1 + e^{-2t}(6\cos t + 4\sin t)$	A1	cao
			$y = 4 + e^{-2t}(6\sin t - 4\cos t)$	A1	cao
				[4]	

Question			Answer	Marks	Guidance
4	(iv)		$\frac{y}{x} \to 4$, so $k = 4$	M1	
				F1	FT wrong constant term(s) and wrong form of expressions for x and y
			$y = 4x \Leftrightarrow 6\sin t - 4\cos t = 4(6\cos t + 4\sin t)$	M1	The exponential term needs to have been cancelled
			$\Leftrightarrow -10\sin t = 28\cos t$		
			$\Leftrightarrow \tan t = -2.8$	A1	Or equivalent e.g. $\tan(t - 0.59) = 4$
			which has infinitely many roots	E1	This can be awarded for a correct justification following a wrong value for $\tan t$ (i.e. can get A0 E1)
				[5]	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

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4758 Differential Equations (Written Examination)

General Comments

The standard of the work presented by most candidates was very high. Solutions were presented neatly and concisely, displaying a sound understanding of, and an ability to apply, the various methods and techniques required. As usual, almost all candidates opted for the questions involving second order linear differential equations and then one of the two questions on first order differential equations. In this series, few candidates made serious attempts at all four questions.

For many candidates, the only loss of marks was in the parts of questions where the request was unfamiliar, requiring some interpretation of results obtained by well-understood methods.

Comments on Individual Questions

1) Second order linear differential equation

(i) Almost all candidates found the general solution in a methodical manner. Some arithmetical slips occurred when solving the simultaneous equations to find the values of the constants for the particular integral.

(ii) Candidates had no problems in applying the boundary conditions to obtain a particular solution of the differential equation.

(iii) Many candidates offered lengthy explanations in which the required answer was identifiable. A simple statement, along the lines of stating that the two given functions each appeared already in the complementary function, was sufficient.

(iv) Again, the method for finding the particular integral for this new differential equation was well executed by the vast majority of candidates. Some made their work more complicated by unnecessarily including terms mentioned in part (iii).

(v) This part of the question required some analysis of the quadratic function appearing in the particular solution obtained in part (iv) and proved difficult even for candidates who had earned full marks up to this point. Most focussed solely on the behaviour of the particular solution for large values of x or for particular values of x. The key feature that candidates needed to identify was that the coefficient of the quadratic was positive, leading to a consideration of the positioning of the graph of the quadratic relative to the x -axis.

2) First order differential equations

This was the least popular choice of question, although a significant number of candidates made an attempt at the first two or three parts and then abandoned the rest in favour of attempting a different question.

(i) The application of Newton's second law to a mechanics problem was well done, followed by the successful use of the method of separation of variables with good use of integration techniques.

(ii) This was almost always correct.

(iii) Most candidates who attempted this part of the question produced excellent solutions using the method of separation of variables. A few opted to find a complementary function and a particular solution and again were usually successful. A minority tried to use the integrating factor method and almost invariably gave up and started on a different question.

(iv) Follow-through was applied to any solution to part (iii) obtained by a legitimate method and this mark was gained by most who attempted it.

(v) This caused problems for some candidates, with a significant minority reverting to the previous model with solution given in part (i). Others, who realised that integration of their solution to part (iv) was required were unsure about limits.

3) First order differential equations

(i) Almost all candidates displayed a good working knowledge of the integrating factor method, with the majority scoring full marks

(ii) There were some excellent sketches of the graph of the particular solution, identifying the key features of oscillations with growing amplitude and two maxima on the x -axis. Many of the other sketches were simply variations on the basic sine or cosine curve, centred on the x -axis and with constant amplitude.

(iii) The need to use the method of separation of variables was identified by most candidates and applied successfully by many. Common errors were $\left(2y^2\right)^{-1} = 2y^{-2}$ and $\frac{1}{2\ln x + c} = \frac{1}{2\ln x} + \frac{1}{c}$

(iv) and less often, $\int y^{-2}\mathrm{d}x = -\frac{1}{3}y^{-3}$.

Although almost all candidates were aware of how to apply Euler's method, this particular example caused more problems than usual. One common cause of error was in manipulating the very small numbers involved and putting the wrong number of zeros after the decimal point.

(v) This routine request was answered correctly by most of the candidates.

4) Simultaneous first order linear differential equations

Candidates are extremely competent at finding the general and particular solutions for x and y from a pair of simultaneous differential equations of this type. As always, it was the last part of the question, which called for some interpretation of the solutions that often led to the loss of a few marks.

(i) The accuracy with which most candidates work in this type of solution is very pleasing. The vast majority obtained the correct second order linear differential equation satisfied by x and solved it successfully. Some made sign errors on the way but were still able to earn most of the available marks.

(ii) This was answered well.

(iii) Again the method was universally known, the only loss of marks being due to slight algebraic slips carried through from earlier parts of the question.

(iv) The first two marks proved very accessible to almost all candidates and follow-through was applied to their solutions in part (iii). The last three marks, however, were gained only by a minority of candidates. Most seemed to have no idea of what was required and did not think to pursue the obvious route of substituting their solutions for x and y into the given expression $y = kx$. Of those who did embark on this route, most did not realise that they could cancel out the exponential term as a non-zero common factor.

GCE Mathematics (MEI)											
			Max Mark	90% cp	a	b	c	d	e	u
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	66	60	53	47	41	34	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	90	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	73	65	57	50	43	36	0
		UMS	100	90	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	66	61	53	46	39	32	0
		UMS	100	90	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	61	54	47	40	34	28	0
		UMS	100	90	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	68	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	90	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	67	63	56	50	44	38	0
		UMS	100	90	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	63	56	49	42	35	29	0
		UMS	100	90	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	66	61	55	49	43	38	0
		UMS	100	90	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	63	56	49	42	35	28	0
		UMS	100	90	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	62	56	50	44	39	34	0
		UMS	100	90	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	52	46	40	34	29	24	0
		UMS	100	90	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	63	55	47	39	32	25	0
		UMS	100	90	80	70	60	50	40	0